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Solar F10.7 Radiation: A Short-Term Statistical Model

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A new method is described for statistically modeling the intensity of the F10.7 decimeter solar radiation for 91-day intervals. This method is based on the analysis of over 12,000 daily F10.7 data values from 1956 to 1989. The model represents this solar flux component as a quasiexponentially correlated Weibull distributed random variable. It is shown that the parameters of the Weibull distribution can be expressed solely as functions of the 91-day average flux value and that this functional representation remains valid during the entire 11-year solar cycle. Modeling the short-term correlation of the daily flux values is accomplished by modifying the recursive procedure for generating an exponentially correlated random variable. The model is accordant with observed F10.7 data, and, therefore, it should be useful for a variety of applications, especially those concerned with density fluctuations of Earth's upper atmosphere.

Introduction

UNDERSTANDING the upper atmosphere has an important bearing on both the design and operation of Space Station Freedom. Controlled motion through the upper atmosphere is impeded by atmospheric drag, and this drag is consequential in other ways as well. First, it largely determines the optimum Space Station altitude, thereby also impinging on Space Shuttle performance in its role of initially constructing and then supplying Space Station Freedom. Because drag directly affects orbital lifetime, it consequently affects the orbital reboost strategy. Drag also generates aerodynamic torques on the Space Station that directly influence vehicle attitude and controllability, and changes in atmospheric density cause these torques to be time varying. Unfortunately, atmospheric density depends on constantly changing factors that are not always well understood and thus difficult to predict. Density is a function of such variables as solar flux, geomagnetic activity, and thermospheric winds. Moreover, density undergoes both regular diurnal and semiannual variations as well as spatial fluctuations in the form of density waves.

Because of these significant and varied effects, a good model of the upper atmosphere is needed for purposes of both the design and operation of Space Station Freedom. A number of models already exist, but these models were not developed with the particular requirements of the Space Station in mind. Consequently, there are some aspects of these models that need more development to match them more closely with Space Station requirements.

Background of Upper Atmosphere Density Variations

Before considering improvements to any of the existing models, it is important to discuss the different causes of variations in the density of the upper atmosphere. One of these causes is the Earth's gravity, which produces a density gradient with altitude. Another contributor to density variations is geomagnetic activity, which leads to short-duration density fluctuations (on the order of days) whose amplitude may vary as much as an order of magnitude. Geomagnetic activity is measured by the geomagnetic index, or a_p, which is conventionally recorded every 3 h. A definite heating does occur during geomagnetic storms, but the heating mechanism is quite complex, making the modeling of this effect very difficult. A third cause is solar radiation, which affects the Earth's atmosphere in several different ways. First among these is the diurnal bulge of the atmosphere, which is an entirely thermal effect caused by direct heating on the sunward side of the Earth during the day and cooling at night. There is also a semiannual variation that is characterized by minima in January and July, and by maxima in April and October.

Another major influence that solar radiation has on density is related to the F10.7 portion of the solar flux. This portion is measured at the 10.7-cm wavelength and is an indicator of the ultraviolet (UV) and extreme ultraviolet (EUV) radiation which heats the upper atmosphere significantly and causes density variations. The F10.7 flux itself does not heat the atmosphere, but it has been shown that the intensity of this radiation, which can easily be measured at the Earth's surface, is correlated with the intensity of the Sun's EUV radiation, which cannot be observed at the Earth's surface. For many purposes, it is useful to consider the time-varying F10.7 solar flux as the composite of a low-frequency background component emanating from the solar disk as a whole and a higher-frequency component originating from highly active regions on the solar disk. These active solar regions vary from day to day and, therefore, their flux is relatively unpredictable. However, the low-frequency background component, obtained by smoothing the raw data over an interval of at least several times the 27-day solar rotation period, manifests a regular variation over a cycle with an average length of just over 11 years. This 11-year mean cycle is generally referred to as the solar cycle, but individual solar-cycle lengths can deviate from the mean by several years. The intensity of cycles also varies significantly, and this unpredictability leads to difficulty in modeling the atmosphere for the time period that Space Station Freedom will be in orbit.

Previous investigations[1-5] of solar activity variations concentrated on characterization of the intermediate-term (months) and long-term (years) behavior. However, now that the F10.7 data have important consequences for the design, operation, and control of Space Station Freedom, the short-term F10.7 fluctuations, on the order of days, have become more significant and thus warrant statistical characterization.

To recapitulate, Space Station altitude, lifetime, attitude, controllability, and reboost strategy are all a function of the density of the upper atmosphere. This density, in turn, is closely correlated with the solar F10.7 radiation since the latter has been shown to correlate well with the solar EUV radiation. And this EUV radiation, emanating from the solar corona and solar flares, contributes greatly to the heating and consequent

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expansion and density variation of the Earth's upper atmosphere.

Purpose of Model Development

The purpose of this study was to develop a model for daily F10.7 fluctuations around a 91-day smoothed mean. Such a model would effectively supplement the long-term modeling and projection of the F10.7 solar flux that has been done previously. The current model,[4,5] developed by NASA Marshall Space Flight Center (MSFC) and used in simulation studies and the Space Station requirements definition, predicts monthly F10.7 and a_p values for a mean solar cycle. The monthly values are generated from data smoothed over 13-month intervals and are given for the 97.5, 50, and 2.5 percentiles of F10.7. The 13-month smoothed values are obtained from two sources: data derived reconditely from smoothed sunspot numbers for the period 1749–1947, and direct F10.7 measurements since 1947. This model is adequate if values are only needed once a month; however, this is not always the case. Currently, to get values at intermediate time intervals, simple interpolation between the closest monthly values is performed. This may be adequate for some long-term applications, but in the process of smoothing over 13 months, a great deal of information on short term fluctuations is lost. In an attempt to quantify this lost detail, an informal study of the variation of a 3-month smoothed value about the 13-month smoothed value was done by MSFC; this showed that, in general, the 3-month values varied within ± 15% of the corresponding 13-month value.

For many specific applications in which a detailed atmospheric model is necessary, a model of short-term F10.7 fluctuations is desired that would replace the current practice of interpolating between 13-month smoothed values. The purpose of this study, therefore, was to determine how the daily F10.7 values vary about a 3-month smoothed average and if these variations could be modeled in a consistent manner across an entire solar cycle.

Development of Statistical Model

The goal of our analysis was to statistically characterize the F10.7 data for intervals of about ¼ year or about 91 days (this interval corresponds to the nominal Space Station reboost frequency). More specifically, we wanted to determine if it was possible to characterize the actual distribution of daily F10.7 values for 91-day intervals as a function of the mean value during each interval. In addition, we wished to model the correlation of the daily data values. This kind of statistical characterization would be of greatest value for Space Station design and operation analyses. Simpler or less detailed characterizations might be of some value, but probably would not be sufficient.

A data base[6] (Ottawa 10.7-cm solar radio flux adjusted to 1 AU) of raw F10.7 measurements was used as a basis for model development. Daily values of the intensity of solar F10.7 radiation are generally available from the year 1947 to the present. From 1947 to 1956 there are, unfortunately, numerous missing data values that cannot all be approximated with interpolated values. However, from late 1956 to the present it is possible to obtain a virtually gapless or uncensored sequence of daily F10.7 values: the small number of missing values in this interval can be filled easily by interpolation from adjacent values. Because this time interval since 1956 is more than 33 years, it is equivalent to three complete, consecutive solar cycles of about 11 years each. From this data, a sequence of 12,162 daily F10.7 values was selected. This number corresponds to the length of three average solar cycles of 4054 days, or 11.1 years, each. This sequence of 12,162 data values, hereafter denoted by Ω, terminated with the value for September 30, 1989.

Initially, we performed some exploratory data analysis on Ω. This analysis, though informative in the usual way, was routine and will not be elaborated here. After several fruitless approaches, we hit on a particular approach that appeared very promising in light of our goal. This approach can be described as follows. For each of 400 successive subsequences of 91 points taken from Ω, we computed estimators of the first three probability weighted moments (PWMs) as discussed by Hosking et al.[7] Please refer to the Appendix for a discussion of PWMs. Each successive subsequence was spaced 30 days later and, thus, overlapped its predecessor for 61 days; this implies that each data point was represented in about three of the 400 subsequences. The 400 subsequences are, therefore, not completely independent, but cover Ω with essentially triple overlap.

Next, a three-dimensional scatter plot was made with each of the three axes representing one of the three PWMs. As mentioned earlier, for each of the 400 subsequences of 91 points, we had computed estimators of the first three PWMs. Thus, these 400 triplets of PWMs were plotted together to help visually manifest any simple functional relationship that might exist among the PWMs. Rather remarkably, this scatter plot showed a very distinct linear relationship among the three PWMs. As shown in Fig. 1, the 400 triplets of PWM values are concentrated along a single straight line in space. This implies that, if one PWM is chosen as the independent variable, then each of the other two PWMs can be determined as a linear function of the independent PWM. Hence, if the first PWM, denoted by PWM1, is chosen to be the independent variable, then PWM2 and PWM3 can each be expressed as linear functions of PWM1. These two linear functions (given in the Appendix) would be determined explicitly from the equation of the straight line that best fits the 400 points of Fig. 1.

This linear dependence among the PWMs is consequential for two reasons. First, since PWM1 is actually identical to the mean,[7] it follows that the linear dependence among the PWMs implies that PWM2 and PWM3 can each be expressed as linear functions of the mean. Secondly, as shown by Hosking et al.,[7] the three PWMs suffice to determine a three-parameter generalized extreme value distribution (GEVD). Thus, all three parameters of the GEVD can readily be determined from the mean, or PWM1, together with the two linear equations relating PWM2 and PWM3 to PWM1. The Appendix discusses this in more detail. If the resulting GEVD for each 91-day subsequence of data provides a statistically realistic representation of the data, then our overall goal would essentially be attained.

This skein of ideas has been successfully implemented. As a result, the distribution of F10.7 data values for any 91-day subsequence of data about a given mean is modeled as a Weibull distribution, which is completely specified by its location, scale, and shape parameters. It should be noted that a Weibull distribution is one of the three possible forms of the

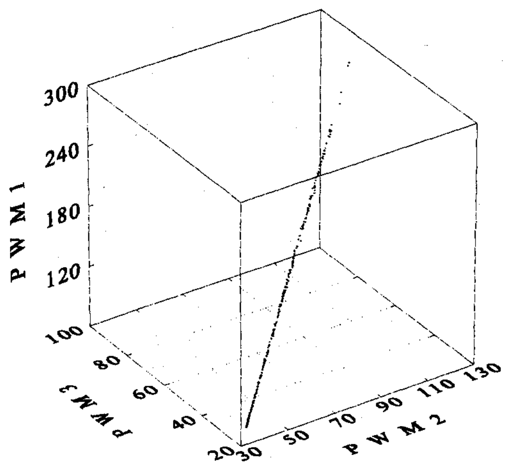

Fig. 1 Scatter plot of the first three PWMs for each of the 400 subsequences of 91-day solar F10.7 data.

GEVD, and its choice was dictated solely by the empirically determined linear relationship among the three PWMs. The distribution function of the Weibull distribution is given by

$$F(x) = 1 - \exp\left[-\left(\frac{x-a}{b}\right)^c\right] \quad (1)$$

where a is the location parameter, b the scale parameter, and c the shape parameter. Functions directly relating the 91-day mean μ (or PWM1) to the three parameters of the appropriate Weibull distribution have been developed and are given in the Appendix. The Weibull distributions obtained from our modeling process have the desirable properties of an abrupt lower cutoff, a single mode, a tapering tailoff at higher values, and a variance that increases with the mean; these characteristics match the empirical data well. Weibull probability density functions are shown in Fig. 2 for several representative values of the F10.7 mean μ.

Correlation

In addition to modeling the distribution of F10.7 data, it is also desirable, as mentioned earlier, to model its correlation. The model just described yields the distribution of daily F10.7 values but, without knowledge of how each value is related to other nearby values in time, a realistic series of modeled values cannot be generated. The autocorrelation function for Ω was determined for lags of up to 200 days, and a graph of this function appears in Fig. 3. The periodicity in this function is due primarily to the Sun's rotation period of about 27 days with respect to the orbiting Earth. It must be recognized that this autocorrelation function is derived from Ω, or the entire set of 12,162 data values; if this function were calculated from data sequences representing only a fraction of a single solar cycle of 11.1 years, the results would look noticeably different.

In creating our model of correlation during 91-day intervals, we have made use of the autocorrelation functions obtained from each of the 400 subsequences of 91-day data. These functions do indeed look different than that depicted in Fig. 3 and, as expected, show a considerable degree of variation among themselves.

Although it might be possible to model the correlation at all time scales, doing so would doubtlessly lead to a rather complex correlation model. Since our primary interest was in modeling the correlation for only a relatively small number of successive daily F10.7 values, we considered only simple correlation models. The most common and probably the simplest correlation model uses an exponentially correlated random variable. However, this model is inextricably associated with normally distributed random variables; whereas the daily F10.7 values follow the Weibull distribution described earlier.

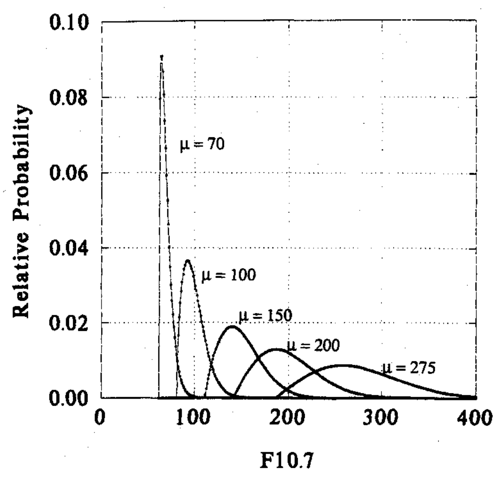

Fig. 2 Weibull probability density functions for representative values of solar F10.7 91-day mean.

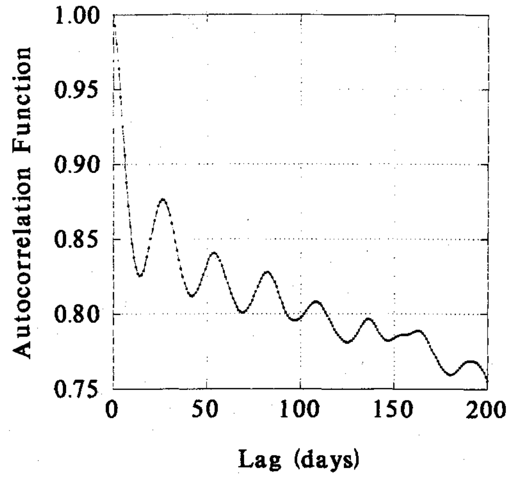

Fig. 3 Autocorrelation function of the solar F10.7 flux over three solar cycles.

To get around this, we create exponentially correlated normal random variables, convert them to correlated uniform random variables, and then transform to the appropriate Weibull distribution. The specific steps involved in this are detailed in the following.

An exponentially correlated, normal random variate ϵ_n with a given correlation constant τ can be generated recursively using the following expression

$$\epsilon_n = C\epsilon_{n-1} + \sqrt{1-C^2}u_n \quad (2)$$

where

$$C = \exp\left(-\frac{\Delta t}{\tau}\right), \qquad \epsilon_0 = 0$$

and when u_n is a normally distributed random variate with zero mean and unit variance. The time between data values Δt should be one day for most applications of this model. From comparisons with 91-day autocorrelation functions of the real data, which describe the correlation throughout the three solar cycles, we settled on a value of $\tau = 11.0$, which describes the correlation reasonably well in any part of the solar cycle. To convert from ϵ_n, which is normally distributed, back to a uniform variate, we use the following approximation for the normal probability distribution[8,9]:

$$U(0,1) = \frac{1}{1 + \exp[-\epsilon_n(a + b\epsilon_n^2)]} \quad (3)$$

where

$$a = \sqrt{\frac{8}{\pi}}, \qquad b = \frac{(4-\pi)}{3\pi}\sqrt{\frac{2}{\pi}}$$

The daily F10.7 value is then generated by converting to a Weibull variate $W(a,b,c)$ from a uniform random variate $U(0,1)$ using the expression for the inverse Weibull:

$$W(a,b,c) = b\{-\ln[U(0,1)]\}^{1/c} + a \quad (4)$$

This process yields correlated values from the appropriate Weibull distribution quickly using a random number generator, but is only intended to generate 91 consecutive values before changing the input value of the 91-day mean. The change of 91-day mean will necessitate solving for the Weibull parameters again, thus slightly altering the Weibull distribution from which we are sampling. It should be noted that, by choosing a simple exponential correlation model, the periodicities associated with the Sun's rotation are smoothed out; these effects could be incorporated into a more sophisticated model if desired.

As a further extension of this model, one could give τ a functional form that varies depending on the location in the solar cycle, as opposed to using a constant value. Using this correlation analysis and the preceding results, every 91-day subsequence of F10.7 data has been modeled as a quasiexponentially correlated Weibull distributed random variable. We have thereby achieved our purpose of characterizing the primary short-term correlation property of the observed data in addition to its distribution properties. This model should therefore be suitable for Space Station applications requiring a detailed model of the F10.7 data and its short-term variation.

Results and Conclusions

As discussed earlier, we have developed a model that requires a 91-day mean F10.7 value as input and yields the three parameters of a Weibull distribution describing the daily F10.7 values within the 91-day interval. The correlation of these values was also modeled, using quasiexponentially correlated random variables, yielding a representative value for the correlation time constant τ across an entire solar cycle. In order to verify the validity of this model, we evaluated the Weibull parameters for various 91-day means and generated sets of 91 modeled daily F10.7 values. These sets were generated by using a Weibull random number generator in conjunction with a quasiexponential correlation algorithm. No significant disparities were observed between the modeled F10.7 values and the actual values, thus corroborating the model. In conclusion, therefore, our model is a significant improvement over previous methods whenever realistic values of F10.7 are desired for comparatively short time intervals. It should be suitable for many Space Station applications.

Appendix: Mathematical Details

Probability Weighted Moments

The most general PWMs of a random variable X with distribution function $F(x) = P(X \leq x)$ are the quantities

$$M_{p,r,s} = E[X^p\{F(X)\}^r\{1 - F(X)\}^s] \quad \text{(A1)}$$

where p, r, and s are real numbers. The quantities $M_{p,0,0}$ ($p = 1, 2, \ldots$) are the usual noncentral moments of X. Following Hosking et al.,[7] we used the moments

$$M_{1,r,0} = E[X\{F(X)\}^r] \quad \text{(A2)}$$

with $r = 0$, 1, and 2. In our discussion, these three PWMs are denoted by PWM1, PWM2, and PWM3, respectively. Unbiased estimators b_r of these moments have been given by Landwehr et al.[10] as

$$b_r = n^{-1} \sum_{j=1}^{n} \frac{(j-1)(j-2)\cdots(j-r)}{(n-1)(n-2)\cdots(n-r)} x_j \quad \text{(A3)}$$

where

$$b_0 = n^{-1} \sum_{j=1}^{n} x_j$$

Parameters of the Generalized Extreme Value Distribution

Hosking et al.[7] have shown that it is possible to obtain the three parameters of the GEVD from the first three PWMs by solving the following three nonlinear equations:

$$b_0 = \xi + \alpha\{1 - \Gamma(1 + \kappa)\}/\kappa \quad \text{(A4)}$$

$$3b_1 - b_0 = \alpha\Gamma(1 + \kappa)(1 - 2^{-\kappa})/\kappa \quad \text{(A5)}$$

$$(3b_2 - b_0)/(2b_1 - b_0) = (1 - 3^{-\kappa})/(1 - 2^{-\kappa}) \quad \text{(A6)}$$

where ξ is the location parameter, α the scale parameter, and κ the shape parameter of the GEVD, and Γ the gamma function. In our case, the data dictate the choice of a Weibull distribution, which is one of the three general forms of the GEVD. The three Weibull parameters are related to the parameters of the GEVD in the following way:

$$a = \alpha/\kappa + \xi \quad \text{(A7)}$$

$$b = \alpha/\kappa \quad \text{(A8)}$$

$$c = 1/\kappa \quad \text{(A9)}$$

An efficient contraction mapping algorithm has been developed for iteratively solving Eq. (A6) for κ. Once κ is known, Eqs. (A5) and (A4) can be solved directly for α and ξ. The Weibull parameters a, b, and c then follow directly from Eqs. (A7–A9).

Linear Relationships Among the Probability Weighted Moments

Using the fitting procedure mention earlier in the text, the linear functions that express PWM2 and PWM3 in terms of PWM1 (the mean) have been determined. These relationships are given by the following equations:

$$\text{PWM2} = 0.445(\text{PWM1}) + 2.26 \quad \text{(A10)}$$

$$\text{PWM3} = 0.280(\text{PWM1}) + 2.25 \quad \text{(A11)}$$

Direct Relationships Between 91-Day Mean and Weibull Parameters

We have developed the following equations, which obviate solving Eqs. (A4–A9) in order to obtain the Weibull parameters from the PWMs of the GEVD. Equations (A12–A14) yield the Weibull parameters immediately, given the 91-day mean μ (PWM1):

$$a = 0.6017\mu + 21.11 \quad \text{(A12)}$$

$$b = 0.4512\mu - 24.24 \quad \text{(A13)}$$

$$c = (\mu - 42.92)/(0.4543\mu - 9.875) \quad \text{(A14)}$$

Acknowledgments

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